# Continuous Bayesian Belief Nets

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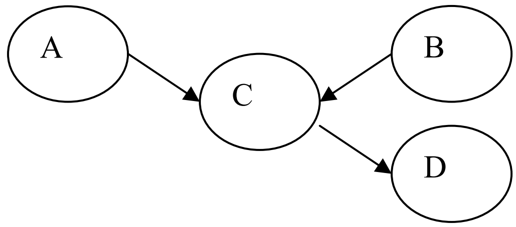
Bayesian belief nets (bbns) are directed acyclic graphs representing high dimensional uncertainty distributions. They are finding rapidly expanding application in modeling of risk and reliability. Their popularity is based on the fact that the influence diagrams capture engineer's intuitive understanding of complex systems, and at the same time serve as user interfaces for sophisticated software systems.

However, for modeling risk in complex systems there are several serious limitations to bbn's as currently used.
(1) Assessment burden: If a node X has K incoming 'influences', where each chance node has M possible outcomes, then the conditional distribution of X must be assessed for each of the $M^K$ input influences. This exorbitant assessment burden can only be reduced by grossly coarse-graining the outputs from nodes and/or introducing simplifying assumptions for the compounding of 'influences'. In practice, chance nodes are often restricted to two possible values.
(2) Continuous chance nodes: Continuous nodes are insupportable unless the joint distribution is given by data and happens to be joint normal. In this case the 'moral graph' can be constructed from the inverse covariance matrix, and updating can be done analytically. Beyond this standard approaches provide no suitable way to represent "influence" for continuous chance variables. To illustrate, consider the situation pictured below with A, B and C having continuous marginal distributions. Should we associate the arrows with correlation coefficients? The permissible correlations are constrained by the marginal distributions – not every correlation is possible with given margins. Further, this graph says that A and B are independent, hence they have correlation zero. This also constrains the correlations $\rho(A,C)$ and $\rho(B,C)$; for example, these cannot both be close to 1. In general not every pair of variables will be connected by arrows in a bbn. In the figure below, (A,D), (B,D) and (AC) are not connected. Although the correlation $\rho(A,B)=0$ is determined by the graph, the correlations $\rho(A,D)$ and $\rho(B,D)$ are not determined. The question whether any set of assessed correlation values for these pairs can be extended to a positive definite correlation matrix is intractable in general. Thus, when correlations are assigned to arrows, it is not in general decidable whether there exists a joint distribution with the specified one-dimensional margins having the specified

correlation structure. In fact this statement holds even if the one-dimensional margins are




all uniform.

(3) The relevant conditional independence structure of a given system cannot always be captured in a single influence diagram. Smith [1990] advocates using several influence diagrams to capture different aspects of a single system.

In [2] the authors introduced an approach to continuous belief nets using vines [1] and the elliptical copula [3]. Influences were associated with conditional rank correlations, and these were realized by (conditional) elliptical copulae. While this approach has some attractive features, notably in preserving some relations between conditional and partial correlation, it also has disadvantages. Foremost among these is the fact that zero (conditional) correlation does not correspond to (conditional) independence under the elliptical copulae.

For this reason we now prefer a different approach which is "copula-free" so long as (1) the chosen copula represents (conditional) independence as zero (conditional) correlation, and (2) conditional correlation is always constant. This approach cannot rely on the equality of partial and conditional correlation, and hence cannot rely on vine transformations to deal with observation and updating. It turns out the some relations between partial and conditional correlation are preserved under the above two assumptions, and that tractable computational rules for updating can be given. Other advantages of the vine – copula approach are retained as well, namely, the vine can be converted into a sampling routine and can deal elegantly with incomplete information.

We show that the elicitation protocol of [2] based on conditional rank correlation can work in a copula-free environment. A unique joint distribution can be determined and sampled based on the protocol which preserves the conditional independence properties of the bbn. Further, this distribution can be updated with observations. The third issue noted above, namely the lack of a single graphical model capturing all relevant conditional independence relations, has not yet yielded, and maybe never will.

**References**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